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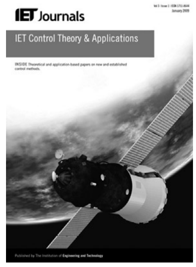

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Robust controller with state-parameter estimation for uncertain networked control system

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Abstract: This study presents the design of an adaptive Kalman filter for networked systems involving random 'sensor delays, missing measurements and packet dropouts'. Two different adaptive filters are considered to estimate unknown parameter vector associated with the system matrices and subsequently the estimation of state and parameters of the system based on the minimisation of square of the output prediction error is adopted in bootstrap manner. An estimator-based robust controller design has been proposed for asymptotic stability of the system whose parameters can vary within a known bound. The effectiveness of the designed algorithms is tested through a numerical example under different cases.

1 Introduction

Feedback control systems wherein the control loops are closed through an uncertain real-time network channel are called networked control systems (NCSs). NCSs appeared recently and have been drawing more and more attentions from researchers working in the fields of systems and control. In a sensor network (SN), independent sensors connected to monitoring stations via uncertain shared communication channel, where data measured by individual sensor are sent to the estimator. With the advent of advanced networking technology a multidisciplinary effort is being taken to develop a networked structure capable to integrate distributed sensors, distributed actuators, and distributed control algorithms over a communication network to make NCS suitable for real-time applications [1].

There are three important aspects in development of such network control system. The first one is to design the networked system (routing control, congestion reduction, efficient data communication etc.) to meet the demands of reliable and concurrent data communication for the control purposes [2]). The second important aspect is to design an estimator-based controller to meet desired performance while the information exchanged among control system components (sensor, controller, actuator etc.) via a shared network is not perfectly reliable one. Another important aspect in NCSs is data packet dropout, which may be potential source of instability and poor performance. The problem of estimating the state of a remote plant based on measurements carried through a lossy network is important applications in NCSs. Owing to limited bandwidth of the channel, the raw sensor measurement data are encapsulated into packets and sent via communication channel, it may suffer random delay. If delay is constant and uniform, it can be treated as constant time-delayed system, which is fairly unrealistic since delays resulting from the network transmission are typically time-varying and it is, in true sense, random by nature. Owing to its random nature, delay must be considered while modelling the NCS. The effect of delay in the system can degrade the performance of control systems designed without considering the delay and can even destabilise the system.

The next uncertainty in NCS is that the data encapsulated into packet may contain noise only (i.e. data packet has no valid measurement) and when such packet is sent to estimator, it is not distinguished from a packet containing valid measurements and no corrective action is taken by the network protocol. This is called missing measurements and usually caused by the unavoidable errors or losses in the transmission.

The another uncertainty in NCS is the loss of data packets because of channel deficiency or data congestion on the channel and cannot be recovered at the receiver side. This is known as data packet loss. Since it occurs randomly, it must be considered separately while modelling the network control system. In fact, data packets through networks suffer not only transmission delay, but also possibly packet dropout. The later is a potential source to destabilise the system and degrade the performance of the NCS. The stability problem of closed-loop NCS in the presence of network delays and data packet dropouts has been focused by several research workers [3–7].

In recent years, much attention has been paid for design and analysis in NCSs. For most of the practical situations, where a plant is connected to a remote-monitoring station via wireless channel, there are two way of uncertainties introduced (i) because of network channel and (ii) because of perturbation in system parameter. If variation in system parameters are known, adaptive Kalman filter can be designed to estimate the system states by considering all possible uncertainties introduced by the network channel. A large number of technical literatures is available related to

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estimation and control in NCSs considering either of above uncertainties ([8–12] and references therein). Uncertainties are represented by discrete-time linear systems with Markovian jumping parameter as described in [13, 14]. In [15], a recursive self-tuning algorithm is adopted to estimate the unknown parameters for a traditional (wired) system, where no uncertainty has been considered in the channel between sensor to estimator. To, the best of our knowledge, no article has been reported for simultaneous state and parameter estimation of unknown system under all three uncertainties introduced by the networked system. In [10], the network systems involving all three uncertainties in 'sensor delays, missing measurements and packet dropouts' have been considered and an adaptive Kalman filter has been considered to estimate the states of time-varying system, whose parameter variations are known. It may be noted that the state estimation algorithm described in [10] is dependent on the state covariance matrix and fails to converge if the open-loop system is unstable and subsequently the trace of the error covariance matrix becomes unbounded. If system matrices are dependent on unknown parameter vector, an adaptive filter for parameter estimation must be combined along with state estimation algorithm in bootstrap manner.

In this paper, the work of [10] is modified by reformulating the expression of error covariance matrix to make sure that the estimator dynamics independent of the state covariance matrix. This, in turn, ensures the convergence of trace of error covariance matrix even though the open-loop system is unstable. Subsequently, an adaptive Kalman filter has been proposed to estimate state and parameter of the NCS in a bootstrap manner whose parameters vary within the known bounds. The innovation process and gradient of innovation process are utilised to estimate the unknown parameters. Furthermore, the system matrices at all points within the convex polyhedron formed with the known bounded parameters may not be stable. In this paper, the existence of robust stabilising controller for the NCS has been considered.

This paper is organised as follows. In section 2, the problem formulation of the system in presence of uncertainty involved between sensor and estimator has been considered. All three uncertainties because of network channel, described above, are modelled in form of an augmented system using their probabilities of occurrence. In Section 3, a general model for the estimator has been presented. Sections 4 and 5 discuss in detail the derivation of modified filter design. In Section 6, parameter estimation algorithm has been described and subsequently, the combined state and parameter estimation algorithm in bootstrap manner is presented in Section 7. Section 8 describes the design of controller for robust stabilisation of uncertain systems with the known bounds of the system parameters. In Section 9, a numerical example has been presented to demonstrate the effectiveness of the proposed algorithm under different conditions. Finally, Section 10 concludes the paper.

2 Problem formulation

2.1 Problem statements

Consider the following parameter-dependent discrete time system

$$x(k+1) = A(\theta)x(k) + B(\theta)w(k) + E(\theta)u(k) \tag{1}$$

$$z(k) = C(\theta)x(k) + v(k) \tag{2}$$

where, $x(k) \in R^n$ is the state vector with $x(k \leq 0) = x(0)$, $z(k) \in R^p$ is the measured output, $u(k) \in R^m$ is the control input, and $w(k) \in R^m$ and $v(k) \in R^p$ are stationary, zero-mean white noise Gaussian sequences with covariance matrices

$$E[\eta(k) \quad \eta^{\mathrm{T}}(k)] = \mathrm{diag}[\Lambda_w(\theta),\ \Lambda_v(\theta)] \quad \text{and}$$
$$E[\eta(k) \quad \eta^{\mathrm{T}}(r)] = 0,\ k \neq r \tag{3}$$

where, $\eta(k) = [w^{\mathrm{T}}(k) \quad v^{\mathrm{T}}(k)]^{\mathrm{T}}$, and $w(k)$ and $v(k)$ are uncorrelated.

Matrices A, B, C, E, Λ_w and Λ_v are continuous functions of an unknown parameter $\theta \in R^s$. It is known that the true value of θ is confined within a compact subset $[a, b]$ of s-dimensional Euclidean space R^s.

The initial conditions satisfy the mean and covariance conditions

$$E[x(0)] = x_0,\ E[(x(0) - x_0)(x(0) - x_0)^{\mathrm{T}}] = p_0 \tag{4}$$

2.2 System model with uncertainties

Systems with mixed uncertainties in sensor delays, missing measurements and packet dropout can be represented by the following model:

If we define $X(k+1) = [x^{\mathrm{T}}(k+1) \quad x^{\mathrm{T}}(k) \quad y^{\mathrm{T}}(k)]^{\mathrm{T}}$ and $W(k) = [w^{\mathrm{T}}(k) \quad v^{\mathrm{T}}(k) \quad v^{\mathrm{T}}(k-1)]^{\mathrm{T}}$ then

$$X(k+1) = A_r(\theta)X(k) + E_r(\theta)u(k) + B_r(\theta)W(k) \tag{5}$$

$$\begin{aligned} y(k) &= C_r(\theta)X(k) + D_r(\theta)[0 \quad I]W(k) \\ &\triangleq C_r(\theta)X(k) + H_r(\theta)W(k) \end{aligned} \tag{6}$$

where, $H_r(\theta) = D_r(\theta)[0 \quad I]$, $I_{2p\times 2p}$ is identity matrix of compatible dimension with $[v^{\mathrm{T}}(k) \quad v^{\mathrm{T}}(k-1)]^{\mathrm{T}}$.

The matrices $\{A_r(\theta), B_r(\theta), C_r(\theta), D_r(\theta), E_r(\theta)\}$ may be represented by using a stochastic binary parameter α_q; $q = 1, 2, 3, 4$ as

$$\begin{aligned} &\{A_r(\theta), B_r(\theta), C_r(\theta), D_r(\theta), E_r(\theta)\} \\ &\triangleq \sum_{q=1}^{4} \alpha_q \{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\} \end{aligned} \tag{7}$$

with, $\sum_{q=1}^{4} \alpha_q = 1$ and $\alpha_q = 0$ *or* 1.

Let $\{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\}$, $q = 1, 2, 3, 4$ denote the four models corresponding to the system with no uncertainty, sensor delay, missing measurement and packet dropout respectively. These system matrices are identified in the following cases.

Case 1: No uncertainty (current measurement, $q = 1$): Measurement equation at the estimator side will be available as

$$\boxed{y(k) = z(k) = C(\theta)x(k) + v(k)}$$

So

$$A_1(\theta) = \begin{bmatrix} A(\theta) & 0 & 0 \\ I & 0 & 0 \\ C(\theta) & 0 & 0 \end{bmatrix}, \quad B_1(\theta) = \begin{bmatrix} B(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$

$$E_1(\theta) = \begin{bmatrix} E(\theta) \\ 0 \\ 0 \end{bmatrix}$$

$C_1(\theta) = [C(\theta) \quad 0 \quad 0]$ and $D_1(\theta) = [I \quad 0]$; $I_{p\times p}$

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is identity matrix of compatible dimension with $v(k)$.

Case 2: One-step sensor delay ($q = 2$): Measurement equation at the estimator side will be available as

$$\boxed{y(k) = z(k-1) = C(\theta)x(k-1) + v(k-1)}$$

So

$$A_2(\theta) = \begin{bmatrix} A(\theta) & 0 & 0 \\ I & 0 & 0 \\ 0 & C(\theta) & 0 \end{bmatrix}, \quad B_2(\theta) = \begin{bmatrix} B(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$E_2(\theta) = \begin{bmatrix} E(\theta) \\ 0 \\ 0 \end{bmatrix}$$

$$C_2(\theta) = [0 \quad C(\theta) \quad 0] \text{ and } D_2(\theta) = [0 \quad I];\ I_{p\times p}$$

is identity matrix of compatible dimension with $v(k)$.

Case 3: Missing measurement (for $q = 3$, the data packet contains no valid measurement, but only noise): Measurement equation at the estimator side will be available as

$$\boxed{y(k) = z(k) = v(k)}$$

So

$$A_3(\theta) = \begin{bmatrix} A(\theta) & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_3(\theta) = \begin{bmatrix} B(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$

$$E_3(\theta) = \begin{bmatrix} E(\theta) \\ 0 \\ 0 \end{bmatrix}$$

$$C_3(\theta) = [0 \quad 0 \quad 0] \text{ and } D_3(\theta) = [I \quad 0];\ I_{p\times p}$$

is identity matrix of compatible dimension with $v(k)$.

Case 4: Packet dropout (for $q = 4$, the whole data packet is lost): At the estimator side only previously stored measurement will be available.

$$\boxed{y(k) = y(k-1)}$$

So

$$A_4(\theta) = \begin{bmatrix} A(\theta) & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \quad B_4(\theta) = \begin{bmatrix} B(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_4(\theta) = \begin{bmatrix} E(\theta) \\ 0 \\ 0 \end{bmatrix}$$

$$C_4(\theta) = [0 \quad 0 \quad I] \text{ and; } D_4(\theta) = [0 \quad 0];\ I_{p\times p}$$

is identity matrix of compatible dimension with $y(k)$.

It is possible to adequately model the system and to assign the probabilities of occurrence to each of above cases, while empirical experimentations and observations have been carried out on NCS under different situations.

Let probability that the system at time index k is given by $\{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\} = \rho_q(k)$, where, $\sum_{q=1}^{4} \rho_q(k) = 1$

So, $\Pr[\alpha_q(k) = 1] = \rho_q(k)$ for $q = 1, 2, 3, 4$.

3 General model of estimator

Considering the state model represented by (5) and (6), the corresponding augmented state estimator dynamics is described as

$$\begin{aligned} X_s(k+1) &= A_s(\theta)X_s(k) + E_s(\theta)u(k) \\ &\quad + G_s(k)[y(k) - C_sX_s(k)] \end{aligned} \tag{8}$$

where, $X_s(k) = [x_s^{\mathrm{T}}(k|k) \quad x_s^{\mathrm{T}}(k-1|k-1)]^{\mathrm{T}}$ and $x_s(k|k) =$ estimated state at time index k to minimise $E[e^{\mathrm{T}}(k)e(k)] = E\{Tr[e(k)e^{\mathrm{T}}(k)]\} = E\{Tr[\Lambda_e(k)]\}$, where $e(k) = x(k) - x_s(k|k)$ and $\Lambda_e(k)$ is the error covariance matrix. The estimator matrices are selected as

$$A_s(\theta) = \begin{bmatrix} A(\theta) & 0 \\ I & 0 \end{bmatrix}, \quad E_s(\theta) = \begin{bmatrix} E(\theta) \\ 0 \end{bmatrix}$$

$$G_s(k) = \begin{bmatrix} K_s(k) \\ 0 \end{bmatrix}$$

and the corresponding time update and measurement update equations are described as

$$x_s(k+1|k) = A(\theta)x_s(k|k) + E(\theta)u(k) \tag{9}$$

$$\begin{aligned} x_s(k+1|k+1) &= x_s(k+1|k) + K_s(k+1) \\ &\quad \times [y(k+1) - C_sX_s(k+1|k)] \end{aligned} \tag{10}$$

where, system matrices $A(\theta)$ and $E(\theta)$ are function of θ and $K_s(k)$ is the Kalman gain matrix at time index k.

Since in the packet dropout case, no new information is received by the estimator, it is treated as an exceptional case, which will be considered separately in the subsequent sections.

4 Modified filter design with sensor delays and missing measurements

In this section, we derive filter equation for the combine cases having no uncertainty, one step sensor delays and missing measurements ($q = 1, 2, 3$).

Let $F = [I_{2n\times 2n} \quad 0]$, $n =$ dimension of state vector x, such that $FX(k) = [x^{\mathrm{T}}(k) \quad x^{\mathrm{T}}(k-1)]^{\mathrm{T}}$. Then $FX(k) - X_s(k) = [e^{\mathrm{T}}(k) \quad e^{\mathrm{T}}(k-1)]^{\mathrm{T}}$.

For $q = 1, 2, 3$, as the right 'block column' of FA_q is 0, it can be shown that $FA_qX(k) = A_sFX(k)$. Similarly, rightmost 'block column' of C_q, $q = 1, 2, 3$ is zero, it can be prove that $C_qX(k) = C_qF^{\mathrm{T}}FX(k)$. Also $FE_q = E_s$. So, if we pre-multiply (5) by F and subtract (8) from it, we obtain

$$\begin{aligned} \zeta(k+1) &= A_s\zeta(k) + [FB_r - G_sH_r]W(k) \\ &\quad - G_s[C_rF^{\mathrm{T}}FX(k) - C_sX_s(k)] \end{aligned} \tag{11}$$

where, $\zeta(k) = FX(k) - X_s(k) = [e^{\mathrm{T}}(k)\ e^{\mathrm{T}}(k-1)]^{\mathrm{T}}$.

Let $\Gamma \triangleq [I \quad 0], I$ of compatible dimension with $e(k)$, such that $e(k) = \Gamma\zeta(k)$. Hence, error covariance can be expressed as

$$\Lambda_e(k) = \Gamma\Lambda_\zeta(k)\Gamma^{\mathrm{T}} \tag{12}$$

Let $\Pr\{$System at time index k is $\{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\}$, given measurement at time index k is

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$y(k)\} \triangleq \pi_q(k)$, for $q = 1, 2$, and 3, where

$$\pi_q(k) = \frac{\rho_q(k)}{\sum_{q=1}^{3} \rho_q(k)}, \quad q = 1, 2, 3 \tag{13}$$

Now for $q = 1, 2, 3, C_s$ is defined as

$$C_s = E_{\alpha_q}[C_r F^{\mathrm{T}}] \quad So, \quad C_s = \sum_{q=1}^{3} \pi_q C_q F^{\mathrm{T}} \tag{14}$$

It may be noted that there are two types of uncertainties (namely process and measurement noises and α_q) in the expression of $\zeta(k)$, (11). If expectation of (11) over stochastic parameter α_q, $q = 1, 2, 3$ is taken and subsequently $E_{\alpha_q}[C_r F^{\mathrm{T}}]$ is replaced by C_s, then (11) can be rewritten as

$$\zeta(k+1) = [A_s - G_s C_s]\zeta(k) + E_{\alpha_q}[FB_r - G_s H_r]W(k) \tag{15}$$

So, augmented error covariance matrix $\Lambda_\zeta(k)$ is modified over [10] and is expressed as

$$\Lambda_\zeta(k) = E_{\alpha_q}[E_W[\zeta(k, \alpha_q)\zeta(k, \alpha_q)^{\mathrm{T}}]]$$

This gives

$$\Lambda_\zeta(k+1) = [A_s - G_s C_s]\Lambda_\zeta(k)[A_s - G_s C_s]^{\mathrm{T}} + E_{\alpha_q}[FB_r - G_s H_r]\Lambda_W[FB_r - G_s H_r]^{\mathrm{T}}$$

or

$$\Lambda_\zeta(k+1) = [A_s - G_s C_s]\Lambda_\zeta(k)[A_s - G_s C_s]^{\mathrm{T}} + \sum_{q=1}^{3} \pi_q [FB_q - G_s H_{]} q \Lambda_W [FB_q - G_s H_q]^{\mathrm{T}} \tag{16}$$

Pre-multiplying by Γ and post multiplying by Γ^{T} to (16), replacing ΓG_s as K_s and then, comparing with (12), error covariance matrix can be expressed as

$$\Lambda_e(k+1) = [\Gamma A_s - K_s C_s]\Lambda_\zeta(k)[\Gamma A_s - K_s C_s]^{\mathrm{T}} + \sum_{q=1}^{3} \{\pi_q \Gamma \Upsilon_q \Lambda_W \Upsilon_q^{\mathrm{T}} \Gamma^{\mathrm{T}}\} \tag{17}$$

The problem of minimising $E[e(k+1)^{\mathrm{T}} e(k+1)]$ may be posed as

$$\min_{K_s} E\{\mathrm{Tr}[\Lambda_e(k+1)]\} \text{ subject to (17).} \tag{18}$$

The necessary condition for the function in (18) to be optimised is given by

$$\frac{\partial}{\partial K_s}(\mathrm{Tr}\{E[\Lambda_e(k+1)]\}) = 0 \tag{19}$$

which gives

$$K_s = \left[\Gamma A_s \Lambda_\zeta(k) C_s^{\mathrm{T}} + \sum_{q=1}^{3} \pi_q \Gamma F B_q \Lambda_W H_q^{\mathrm{T}}\right] \times \left[C_s \Lambda_\zeta(k) C_s^{\mathrm{T}} + \sum_{q=1}^{3} \pi_q H_q \Lambda_W H_q^{\mathrm{T}}\right]^{-1} \tag{20}$$

If we put $G_s = 0$ in (16), we obtain one step prediction equation for error covariance in augmented form

$$\Lambda_\zeta(k+1|k) = A_s \Lambda_\zeta(k|k) A_s^{\mathrm{T}} + \sum_{q=1}^{3} \pi_q F B_q \Lambda_W B_q^{\mathrm{T}} F^{\mathrm{T}} \tag{21}$$

Setting $A_s = I$, and $B_q = 0$ in (20) one can obtain Kalman filter gain as

$$K_s = \Gamma \Lambda_\zeta(k) C_s^{\mathrm{T}} \left[C_s \Lambda_\zeta(k) C_s^{\mathrm{T}} + \sum_{q=1}^{3} \pi_q H_q \Lambda_W H_q^{\mathrm{T}}\right]^{-1} \tag{22}$$

Since, $K_s = \Gamma G_s$, G_s may be selected as $[K_s^{\mathrm{T}} \quad 0]^{\mathrm{T}}$. Again setting $A_s = I$ and $B_q = 0$ in (16), one can obtain measurement update equation for augmented error covariance matrix as

$$\Lambda_\zeta(k|k) = [I - G_s C_s]\Lambda_\zeta(k)[I - G_s C_s]^{\mathrm{T}} + \sum_{q=1}^{3} \{\pi_q G_s H_q \Lambda_W H_s^{\mathrm{T}} G_q{}^{\mathrm{T}}\} \tag{23}$$

5 Filter design with packet dropout

When packet dropout occurs, no new measurement information is received. That is, if kth sampled data packet is lost during transmission, the network protocol at the receiver side does not sense the arrival of any data packet at the corresponding sampled instant and only previously stored data in the buffer is available to the estimator. So, at kth sampled index, measurement data are taken as $y(k) = y(k-1)$. Since, $y(k-1)$ has already been processed, so, at kth sampled index, estimator should proceed with the predicted state and error covariance based on past estimates and skip the measurement update equation.

Predicted state for this case can be obtained by setting $G_s = 0$ in (8)

$$X_s(k+1) = A_s X(k) + E_s u(k) \tag{24}$$

Pre-multiplying (5) by F for $r = q = 4$ (packet dropout case) and subtracting (24) from it, results augmented error vector, $\zeta(k+1)$, and subsequently, augmented error covariance matrix can be expressed as

$$\Lambda_\zeta(k+1) = A_s \Lambda_\zeta(k) A_s^{\mathrm{T}} + F B_4 \Lambda_W B_4^{\mathrm{T}} F^{\mathrm{T}} \tag{25}$$

6 Parameter estimation

If true value of θ were known, the filtering algorithm would be proposed as conventional Kalman Filter based on above filter derivation. Since, from Section 2.1, system matrices are continuous function of unknown parameter θ, an estimate of the unknown parameter θ is used to identify the system matrices as discussed in [15]. Based on measurement information, innovation sequence and its gradient can be expressed as

$$\tilde{y}(k) = y(k) - C_s X_s(k|k-1) \tag{26}$$

$$\partial_j \tilde{y}(k) = -(\partial_j C_s) X_s(k|k-1) - C_s \partial_j X_s(k|k-1) \tag{27}$$

where $\tilde{y}(k)$ and $\partial_j \tilde{y}(k) = \frac{\partial \tilde{y}(k)}{\partial \theta_j}$ are called innovation process and gradient of innovation process respectively. Existence

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of gradient innovation sequence has been discussed in [15]. $\tilde{y}(k)$ is also termed as output prediction error. Parameter estimation criterion is, now, posed as minimisation of mean-square out prediction error

$$\min\{E\|\tilde{y}(k)\|^2\}, \quad \text{for all time index } k.$$

Let the mean-square output prediction error at time index k, given the measurement up to time index k, be

$$\Xi(\theta) = E(\|\tilde{y}(k)\|^2|\{y_1, y_2, \ldots, y_n\}) \tag{28}$$

For $\Xi(\theta)$ to be minimum, for all k

$$2 * E\left(\sum_{j=1}^{s}\left(\frac{\partial\tilde{y}(k)}{\partial\theta_j}\right)^{\mathrm{T}}\tilde{y}(k)|\{y_1, y_2, \ \ldots, \ y_n\}\right) = 0 \tag{29}$$

where, j denotes the jth component of the vector $\theta \in R^s$.

Note: ∂_j *denotes for* $\frac{\partial}{\partial\theta_j}$ *which stands for derivative with respect to jth component of the* θ.

Condition given by (29) is satisfied iff the estimate of the θ approaches to its true value. To achieve the above objective of self-tuning adaptive-filtering, parameter estimation algorithm based on gradient of innovation sequence and its gradient is posed as

For $j = 1, 2, \ldots, s$, all components of $\theta_{1\times s}$ are given by

$$\theta_j'(k) = \theta_j(k-1) - \gamma_k\left(\partial_j\tilde{y}(k)\right)^{\mathrm{T}}\tilde{y}(k) \tag{30}$$

$$\begin{aligned} \theta_j(k) &= b_j; \quad \text{if } \theta_j'(k) \geq b_j \\ &= \theta_j'(k); \quad \text{if } a_j \leq \theta_j'(k) \leq b_j \\ &= a_j; \quad \text{if } \theta_j'(k) \leq a_j \end{aligned} \tag{31}$$

where, γ_k is the positive-conversing sequence. $\partial_j\tilde{y}(k)$ and $\tilde{y}(k)$ is calculated recursively using (26) and (27).

7 State and parameter estimation algorithm

Adaptive filter for combined state and parameter estimation with aforementioned uncertainties is implemented in two state Markov Chain, where, the first case stands for no packet dropout (combination of $q = 1, 2, 3$) and second for packet dropout case ($q = 4$). Since in the case of packet dropout, no new data packet is received, we can say if $y(k) = y(k-1)$, then it is certainly packet dropout case and if $y(k) \neq y(k-1)$, it is the case of $q = 1, 2, 3$.

Case 1*:* $y(k) \neq y(k-1)$

Pr{System at time index k is $\{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\}$, given that measurement at time index k is $y(k)\} \triangleq \pi_q(k)$, for $q = 1, 2, 3$, where $\pi_q(k)$ is given by (13)

Pr{System at time index k is $\{A_4(\theta), B_4(\theta), C_4(\theta), D_4(\theta), E_4(\theta)\}$, given that measurement at time index k is $y(k)\} \triangleq \pi_4(k) = 0$.

Case 2*:* $y(k) = y(k-1)$

Pr{System at time index k is $\{A_q(\theta), B_q(\theta), C_q(\theta), D_q(\theta), E_q(\theta)\}$, given that measurement at time index k is $y(k)\} \triangleq \pi_q(k) = 0$, for $q = 1, 2, 3$.

Pr{System at time index k is $\{A_4(\theta), B_4(\theta), C_4(\theta), D_4(\theta), E_4(\theta)\}$, given that measurement at time index k is $y(k)\} \triangleq \pi_4(k) = 1$.

Based on above probabilities, the filter algorithm for combined state estimation and parameter estimation is implemented in a bootstrap manner as given below.

7.1 Initialisations

System matrices $\{A(\theta), B(\theta), E(\theta), C(\theta)\}$ and noise covariances $\{\Lambda_w(\theta), \Lambda_v(\theta)\}$ should be known as continuous function of the parameter vector θ, whose true value is confined within a known bound $[a, b]$. Its initial value is given by $\theta(0)$.

$$A = A(\theta(0)), \quad A_s = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix} \quad \text{and} \quad \frac{\partial A_s}{\partial\theta_j} = \begin{bmatrix} \frac{\partial A}{\partial\theta_j} & 0 \\ 0 & 0 \end{bmatrix}$$

Error covariance matrix $= P_0$, and $E[x(0)] = x_0, x_s(0) = 0$. Then, $\Lambda_\zeta(0) = \begin{bmatrix} P_0 & P_0 \\ P_0 & P_0 \end{bmatrix}$.

Gradient of initial error covariance $\frac{\partial P_0}{\partial\theta_j}$ is taken as zero. So

$$\frac{\partial\Lambda_\zeta(0)}{\partial\theta_j} = 0, \ \text{for all components of } \theta \ (j = 1, 2, \ldots, s)$$

$X_s(0) = [x_s^{\mathrm{T}}(0) \quad x_s^{\mathrm{T}}(0)]^{\mathrm{T}} = [0 \quad 0]^{\mathrm{T}}$.

7.2 Kalman filter (algorithmic steps)

1. *State Prediction*

$$x_s(k|k-1) = Ax_s(k-1|k-1) + Eu(k-1) \tag{32}$$

$$X_s(k|k-1) = [x_s^{\mathrm{T}}(k|k-1) \quad x_s^{\mathrm{T}}(k-1|k-1)]^{\mathrm{T}} \tag{33}$$

Gradient of predicted state

$$\partial_jx_s(k|k-1) = A\partial_jx_s(k-1|k-1) + (\partial_jA)x_s(k-1|k-1) + \partial_j(Eu(k-1)) \tag{34}$$

$$\partial_jX_s(k|k-1) = [\partial_jx_s^{\mathrm{T}}(k|k-1) \quad \partial_jx_s^{\mathrm{T}}(k-1|k-1)]^{\mathrm{T}} \tag{35}$$

2. *Predicted error covariance matrix and its gradient*

$$\Lambda_\zeta(k|k-1) = A_s\Lambda_\zeta(k-1|k-1)A_s^{\mathrm{T}} + \sum_{q=1}^{3}\pi_qFB_q\Lambda_WB_q^{\mathrm{T}}F^{\mathrm{T}} \tag{36}$$

$$\begin{aligned} \partial_j\Lambda_\zeta(k|k-1) = {} & (\partial_jA_s)\Lambda_\zeta(k-1|k-1)A_s^{\mathrm{T}} \\ & + A_s(\partial_j\Lambda_\zeta(k-1|k-1))A_s^{\mathrm{T}} \\ & + A_s\Lambda_\zeta(k-1|k-1)(\partial_jA_s)^{\mathrm{T}} \\ & + \sum_{q=1}^{3}\pi_qF[(\partial_jB_q)\Lambda_WB_q^{\mathrm{T}} \\ & + B_q(\partial_j\Lambda_W){B_q}^{\mathrm{T}} + B_q\Lambda_W(\partial_jB_q)^{\mathrm{T}}]F^{\mathrm{T}} \end{aligned} \tag{37}$$

3. *Kalman gain matrix*: If $y(k) = y(k-1)$, (packet dropout) then

$$K_s(k) = 0 \tag{38}$$

$$\partial_jK_s(k) = 0 \tag{39}$$

else

$$K_s(k) = \Gamma\Lambda_\zeta(k|k-1)C_s^{\mathrm{T}}\Pi^{-1} \tag{40}$$

$$\begin{aligned} \partial_jK_s(k) = {} & \Gamma(\partial_j\Lambda_\zeta(k|k-1))C^{\mathrm{T}}\Pi^{-1} + \Gamma\Lambda_\zeta(k|k-1) \\ & \times(\partial_jC_s^{\mathrm{T}})\Pi^{-1} + \Gamma\Lambda_\zeta(k|k-1)C_s^{\mathrm{T}}\Pi^{-1}(\partial_j\Pi)\Pi^{-1} \end{aligned} \tag{41}$$

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where, $\Pi = C_s\Lambda_\zeta(k|k-1)C_s^{\mathrm{T}} + \sum_{q=1}^{3}\pi_q H_q \Lambda_W {H_q}^{\mathrm{T}}$ and

$$\partial_j\Pi = (\partial_j C_s)\Lambda_\zeta(k|k-1)C_s^{\mathrm{T}} + C_s(\partial_j\Lambda_\zeta(k|k-1))C_s^{\mathrm{T}} + C_s\Lambda_\zeta(k|k-1)(\partial_j {C_s}^{\mathrm{T}}) + \sum_{q=1}^{3}\pi_q H_q(\partial_j\Lambda_w)H_q$$

4. *Measurement update of state*

$$\tilde{y}(k) = y(k) - C_s X_s(k|k-1) \tag{42}$$

$$\partial_j\tilde{y}(k) = -(\partial_j C_s)X_s(k|k-1) - C_s\partial_j X_s(k|k-1) \tag{43}$$

$$x_s(k|k) = x_s(k|k-1) + K_s(k)\tilde{y}(k) \tag{44}$$

$$\partial_j x_s(k|k) = \partial_j x_s(k|k-1) + (\partial_j K_s(k))\tilde{y}(k) + K_s(k)(\partial_j\tilde{y}(k)) \tag{45}$$

5. *Covariance matrix update*

$$G_s = [K_s^{\mathrm{T}} \quad 0]^{\mathrm{T}} \tag{46}$$

$$\partial_j G_s = [\partial_j K_s^{\mathrm{T}} \quad 0]^{\mathrm{T}} \tag{47}$$

$$\Lambda_\zeta(k|k) = [I - G_s C_s]\Lambda_\zeta(k+1|k)[I - G_s C_s]^{\mathrm{T}} + \sum_{q=1}^{3}\pi_q G_s H_q \Lambda_W H_q^{\mathrm{T}} G_s^{\mathrm{T}} \tag{48}$$

$$\begin{aligned}\partial_j\Lambda_\zeta(k+1|k+1) = &-[(\partial_j G_s)C_s + G_s\partial_j C_s]\Lambda_\zeta(k+1|k) \\ &\times [I - G_s C_s]^{\mathrm{T}} - [I - G_s C_s]\Lambda_\zeta \\ &\times (k+1|k)[(\partial_j G_s)C_s + G_s\partial_j C_s]^{\mathrm{T}} \\ &+ [I - G_s C_s](\partial_j\Lambda_\zeta(k+1|k)) \\ &\times [I - G_s C_s]^{\mathrm{T}} \\ &+ \partial_j\left(\sum_{q=1}^{3}\pi_q G_s H_q \Lambda_W H_q^{\mathrm{T}} G_s^{\mathrm{T}}\right)\end{aligned} \tag{49}$$

6. *Parameter update*

For $j = 1, 2, \ldots, s$, all components of $\theta_{1\times s}$ are updates as

$$\theta_j'(k) = \theta_j(k-1) - \gamma_k(\partial_j\tilde{y}(k))^{\mathrm{T}}\tilde{y}(k) \tag{50}$$

$$\begin{aligned}\theta_j(k) &= b_j; \quad \text{if } \theta_j'(k) \ge b_j \\ &= \theta_j'(k); \quad \text{if } a_j \le \theta_j'(k) \le b_j \\ &= a_j; \quad \text{if } \theta_j'(k) \le a_j,\end{aligned} \tag{51}$$

where, γ_k is the positive conversing sequence. So

$$A = A(\theta(k)) \quad \text{and} \quad A_s = \begin{bmatrix} A & 0 \\ I & 0 \end{bmatrix}$$

The above steps are repeated recursively for the estimation of states and parameters at every time index k.

Remark 1: The stability analysis of combined state and parameter estimation-based state feedback controller under network-induced uncertainties is not immediate or straightforward. In the present problem, the knowledge of estimated parameter and states is utilised in compensator (controller) design to improve the system performance. Sufficient conditions for the existence of stabilising common controller for a family of plants via convex polyhedron approach in LMI framework based on Lyapunov function with the estimated states are discussed in next section.

8 Robust stabilisation of NCS with parameteric uncertainties

The above proposed algorithm is used to estimate the states and parameters of the system based on innovation sequence in measurement under network induced uncertainties. An estimator based robust controller is designed to stabilise the system at every point of the convex polyhedron of parameters bound. The Fig. 1 shows the block diagram representation of uncertain NCS with state and parameter estimation algorithm along with the controller where, controller and actuator are connected directly through a reliable channel.

Since network-induced uncertainties have already been considered in design of estimator. However, the dynamics of NCS is governed by the combined effect of estimated state and gradient of estimated state, those are represented as

$$x_s(k+1|k) = Ax_s(k|k) + Eu(k) \tag{52}$$

$$\partial_j x_s(k+1|k) = A\partial_j x_s(k|k) + (\partial_j A)x_s(k|k) + (\partial_j E)u(k) + E\partial_j u(k) \tag{53}$$

State feedback controller law is expressed as

$$u(k) = K_c x_s(k|k) \tag{54}$$

$$\partial_j u(k) = K_c \partial_j x_s(k|k) \tag{55}$$

where K_c is controller gain vector. So, closed-loop systems are given as

$$x_s(k+1|k) = (A + EK_c)x_s(k|k) \tag{56}$$

$$\partial_j x_s(k+1|k) = (A + EK_c)\partial_j x_s(k|k) + [(\partial_j A) + (\partial_j E)K_c]x_s(k|k) \tag{57}$$

It may be noted that each element of the matrix A in (56) and (57) is a continuous function over a compact set with its lower and upper bounds. A sufficient condition for the stability of such system at all points within the polyhedron (Fig. 2) formed with the corner matrices $A_i, i = 1, 2, \ldots, s$ is stated below [16]

Theorem 1: The sufficient condition for the system described by (56) and (57) to be asymptotically stable at all points on the polyhedron vertices with memoryless state

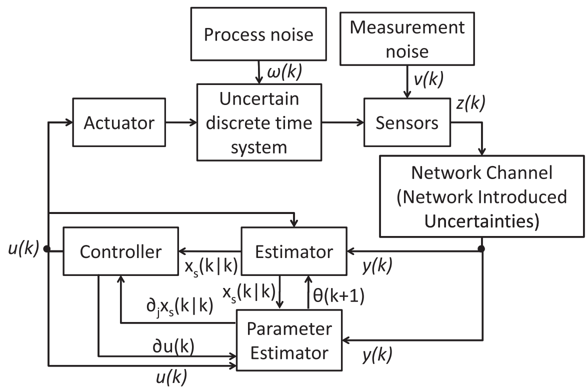

Fig. 1 *NCS system with estimator-based controller (network-induced uncertainties are considered only on sensor to estimator)*

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feedback controller gain $K_c = \Phi\Psi^{-1}$ is that there exist $\Psi = \Psi^{\mathrm{T}} > 0$ and Φ for given scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$ satisfying the following LMIs (see (58))
where, $A_i, i = 1, 2, \ldots, 2^s$ is system matrix at ith coordinate of the polyhedron.

Proof: Let the Lyapunov functional be

$$V(\xi(k|k), k) = x_s^{\mathrm{T}}(k|k)Px_s(k|k) + (\partial_j x_s^{\mathrm{T}}(k|k))Q(\partial_j x_s(k|k)) \tag{59}$$

where $P = P^{\mathrm{T}} > 0, Q = Q^{\mathrm{T}} > 0$ and $\xi(k) = [x_s^{\mathrm{T}}(k)\ \ \partial_j x_s^{\mathrm{T}}(k)]^{\mathrm{T}}$. Then, we have

$$\begin{aligned}\Delta V(\xi(k|k), k) &= V(\xi(k+1|k), k+1) - V(\xi(k|k), k)\\ &= x_s^{\mathrm{T}}(k+1|k)Px_s(k+1|k) - x_s^{\mathrm{T}}(k|k)Px_s(k|k)\\ &\quad + (\partial_j x_s^{\mathrm{T}}(k+1|k))Q(\partial_j x_s(k+1|k))\\ &\quad - (\partial_j x_s^{\mathrm{T}}(k|k))Q(\partial_j x_s(k|k))\end{aligned} \tag{60}$$

Substituting (52)–(55) into (60) and rearranging

$$\Delta V(\xi(k), k) = \xi^{\mathrm{T}}(k)\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}\xi(k) \tag{61}$$

where

$$\begin{aligned}\Sigma_{11} &= -P + (A + EK_c)^{\mathrm{T}}P(A + EK_c)\\ &\quad + [\partial_j A + (\partial_j E)K_c]^{\mathrm{T}}Q[\partial_j A + (\partial_j E)K_c]\\ \Sigma_{12} &= [\partial_j A + (\partial_j E)K_c]^{\mathrm{T}}Q(A + EK_c)\\ \Sigma_{21} &= (A + EK_c)^{\mathrm{T}}Q(\partial_j A + (\partial_j E)K_c) \text{ and}\\ \Sigma_{22} &= (A + EK_c)^{\mathrm{T}}Q(A + EK_c) - Q\end{aligned}$$

For asymptotic stability of the estimated states described by (52)–(55), $\Delta V(\xi(k), k) < 0$ or alternatively

$$\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix} < 0 \tag{62}$$

Applying Schur complements to (62) and substituting $P = \epsilon_1\Psi^{-1}$ and $Q = \epsilon_2\Psi^{-1}$, (62) can be expressed as

$$\begin{bmatrix}-\epsilon_1\Psi^{-1} & * & * & *\\ 0 & -\epsilon_2\Psi^{-1} & * & *\\ \partial_j A + (\partial_j E)K_c & (A + EK_c) & -\epsilon_2^{-1}\Psi & *\\ (A + EK_c) & 0 & 0 & -\epsilon_1^{-1}\Psi\end{bmatrix} < 0 \tag{63}$$

where, scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$ are tuning parameters.

Pre- and post-multiplying (63) by diag$[\Psi, \Psi, I, I]$ and then, substituting $K_c\Psi = \Phi$, we obtain the LMI (58). □

Remark 2: A common controller K_c is designed by solving a set of LMI (58) at the coordinate points of the convex

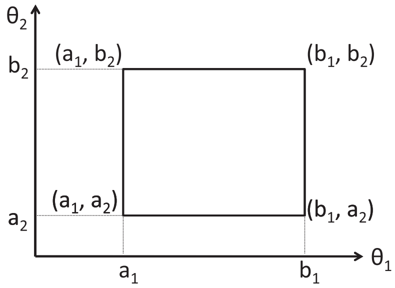

Fig. 2 *Convex polygon of two parameters with $\theta_1 \in (a_1, b_1)$ and $\theta_2 \in (a_2, b_2)$*

polyhedron formed by the parameters bound as shown in Fig. 2, which ensures the stability of the closed-loop system in presence of uncertainties but without any any guarantee of good performance. However, the positive scalars ϵ_1 and ϵ_2 can be tuned judiciously to improve the performance of the system.

9 Numerical examples

In this section, the effectiveness of the algorithm described in Sections 7.2 has been illustrated by a numerical example under different cases.

The system matrices for the state-space model described by (1) and (2) are given as

$$A(\theta) = \begin{bmatrix}1.724\theta & -0.743583\theta\\ \theta & 0\end{bmatrix}, \quad B(\theta) = \begin{bmatrix}1\\ 0.5\end{bmatrix},$$
$$E(\theta) = \begin{bmatrix}1\\ 1\end{bmatrix}, \quad \text{and} \quad C(\theta) = [1 \quad 1]$$

where θ is the unknown parameter whose true value is confined in a compact set [0.6, 1.4]. Initial estimate of θ is assumed to be $\theta(0) = 1$ and positive sequence $\gamma_k = \gamma_0/k$,

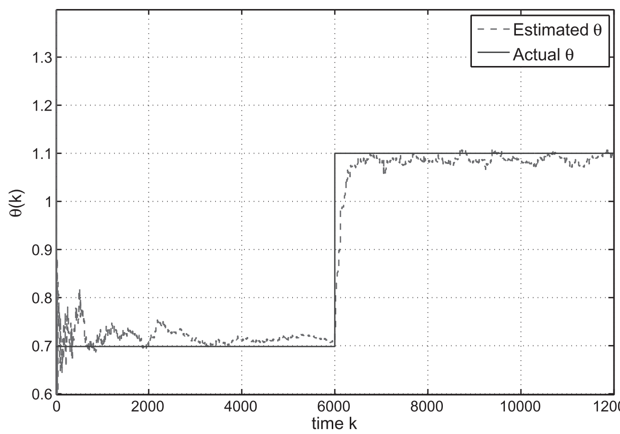

Fig. 3 *Estimated θ (broken line) and its true value (solid line) without controller ($\theta = 0.7$ for first 6000 sample index then changed to 1.1)*

$$\Omega_i = \begin{bmatrix}-\epsilon_1\Psi & * & * & *\\ 0 & -\epsilon_2\Psi & * & *\\ (\partial_j A)\Psi + (\partial_j E)\Phi & A_i\Psi + E\Phi & -\epsilon_2^{-1}\Psi & *\\ A_i\Psi + E\Phi & 0 & 0 & -\epsilon_1^{-1}\Psi\end{bmatrix} < 0 \tag{58}$$

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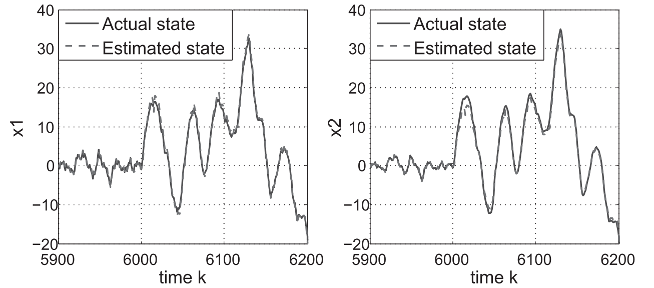

Fig. 4 *State responses without controller (actual value of θ = constant and perturbation is given at 6000 time index)*

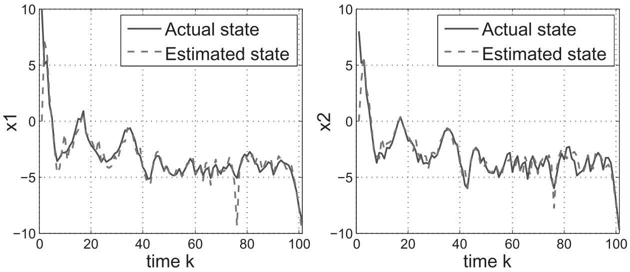

Fig. 5 *State responses without controller with θ varying randomly*

where $\gamma_0 = 0.4$. Process and measurement noise covariance matrices are given as $\Lambda_w = 1$, and $\Lambda_v = 0.25$, respectively.

Gradients of system matrices are given by

$$\frac{\partial A}{\partial \theta_j} = \begin{bmatrix} 1.7240 & -0.7488 \\ 1 & 0 \end{bmatrix} \quad \text{and}$$

$$\frac{\partial B}{\partial \theta_j} = \frac{\partial C}{\partial \theta_j} = \frac{\partial E}{\partial \theta_j} = \frac{\partial \Lambda_w}{\partial \theta_j} = \frac{\partial \Lambda_v}{\partial \theta_j} = 0$$

Initial values $x(0) = [10 \quad 8]^{\mathrm{T}}$, $P_0 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$ and $x_s(0) = [0 \quad 0]^{\mathrm{T}}$. Probabilities of uncertainties for the NCS because of network channel are $\rho_1 = 0.8$, $\rho_2 = 0.02$, $\rho_3 = 0.09$ and $\rho_4 = 0.09$. Initial gradient of state and covariance are

$$\frac{\partial x_s(0)}{\partial \theta_j} = [0 \quad 0]^{\mathrm{T}} \quad \text{and} \quad \frac{\partial P_0}{\partial \theta_j} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Simulation results are considered for the following cases:

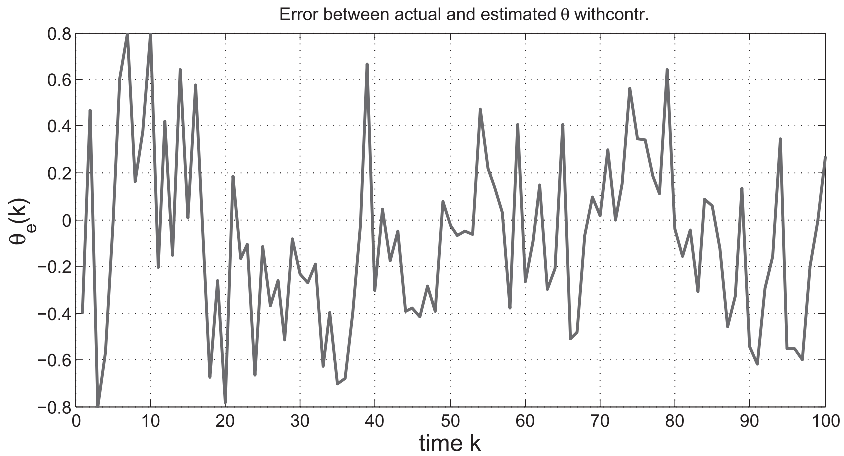

Fig. 6 *Error between estimated θ and true value of θ without controller (θ is varying randomly)*

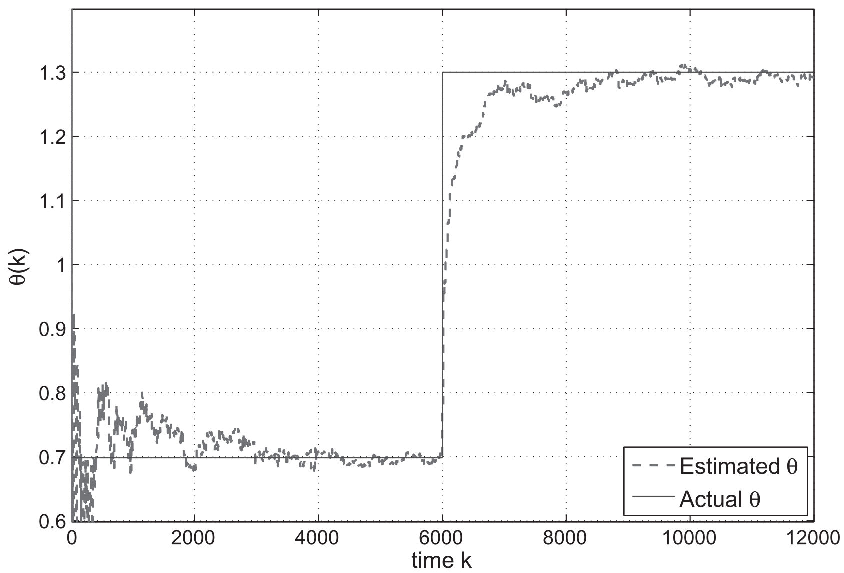

Fig. 7 *Estimated θ (broken line) and its true value (solid line) with controller (θ = 0.7 for first 6000 sample index then changed to 1.3)*

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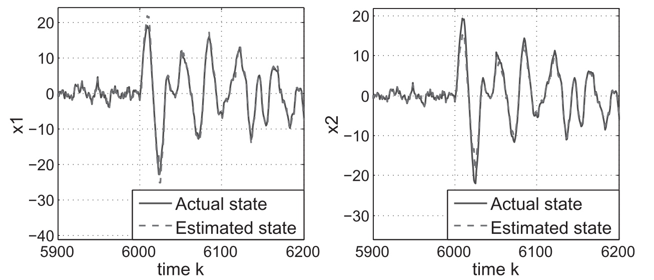

Fig. 8 *State responses with controller (actual value of $\theta =$ constant and perturbation is given at 6000 time index*

9.1 Without controller (u(k)=0)

For constant parameter (θ): We have considered initially the true value of θ is 0.7 and after 6000 iteration its value is changed to 1.1.

Fig. 3 shows the effectiveness of the parameter estimation algorithm. Actual and estimated states under the parametric uncertainties are shown in Fig. 4.

Parameter (θ) changing randomly: Actual value of θ may have any value in the bound [0.6, 1.4] and the estimated states and parameters responses are shown in Figs. 5 and 6.

9.2 With controller

For the present case, system matrix $A(\theta)$ is function of scalar θ, there are two extreme point system matrices. Tuning parameters ϵ_1 and ϵ_2 are determined by hit and trial method to obtain feasible solution of LMIs ($\Omega_i < 0, i = 1, 2$) given by (58), at two extreme values of the system parameters (θ). Solution of the set of two LMIs for the tuned parameters $\epsilon_1 = 0.0016$ and $\epsilon_2 = 0.00018$ are given as

$$K_c = [1.5894 \quad -2.6459]$$

$$\Psi = \begin{bmatrix} 14.4911 & 13.2922 \\ 13.2922 & 12.2848 \end{bmatrix}$$

System responses with controller are given following cases.

For constant parameter (θ): We have considered initially the true value of θ is 0.7 and after 6000 iteration its value is changed to 1.1.

Since, the system matrix at $\theta = 1.3$ is unstable (eigenvalue are $\{1.1206 + j0.0364, \ 1.1206 - j0.0364\}$, the designed controller stabilises the NCS based on estimated states as illustrated in Figs. 7 and 8.

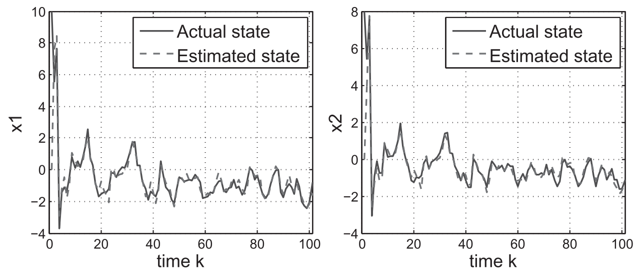

Fig. 9 *State responses with controller (true value of θ is varying randomly)*

Parameter (θ) changing randomly: Actual value of θ may have any value in the bound $[0.6, 1.4]$ and the estimated states and parameters responses with the controller are given by Figs. 9 and 10. It can be seen from Fig. 9, that the designed controller stabilises the NCS under the parametric perturbation in the given bound [0.6 1.4]. Since, the system parameter θ is estimated based on gradient in measurement innovation, the random change in actual value of θ reflects slowly on estimated θ (see, Figs. 3, 6, 7 and 10).

Fig. 11 shows the variation in trace of error covariance under the case of (i) parameter (θ) is constant and (ii) parameter (θ) is randomly changing. It can be noted that the packet dropout time indices appear as spikes in variation of trace of error covariance matrix.

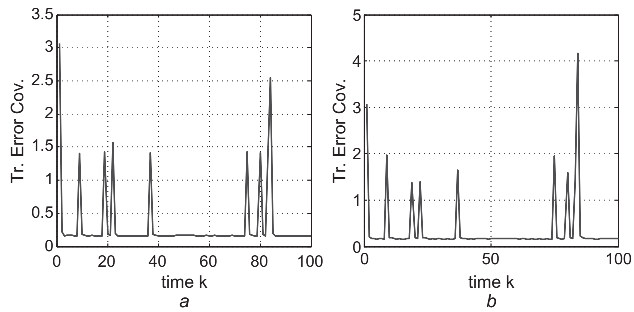

Fig. 11 *Trace of error covariance*

a θ is constant

b θ is randomly changing

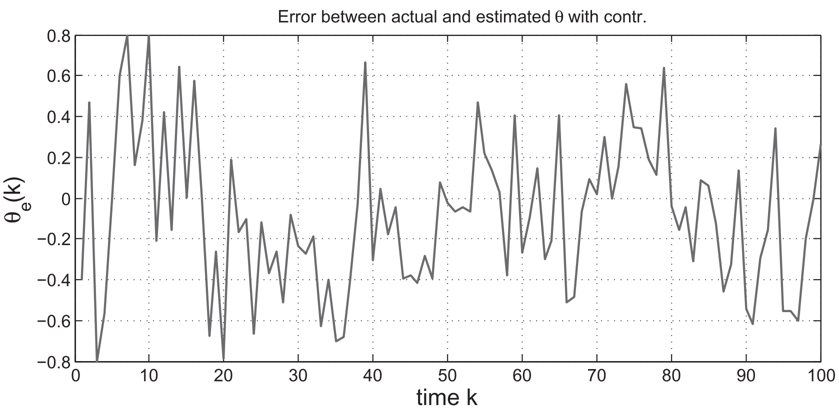

Fig. 10 *Error between estimated θ and true value of θ with controller (θ is varying randomly)*

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10 Conclusions

In this paper, design of adaptive Kalman filter has been presented for the network control systems involving random sensor delays, missing measurements and packet dropouts. Adaptive Kalman filter gain is designed for a parameter-dependent system where the variations of system parameters are unknown, but its lower and upper bounds around the nominal values of parameters are assumed to be known a priori. Simultaneous state and parameter estimation algorithms are adopted based on innovation in measurement under all three uncertainties in measurements that have some a priori knowledge of probabilities of occurrence. A robust controller is designed to stabilise the NCS under the structure parameters perturbation within the given bound. The design of controller uses the vertices of system matrices corresponding to extreme point values of parameters. The state and parameter estimation algorithms are combined in bootstrap manner to implement the control, which in turn stabilises the NCS.

In the present work, we have considered the possibility of uncertainties from sensor to estimator only. Here, we have assumed that the controller is near by the actuator and the generated control signal is available to the actuator as well as estimator without any uncertainties. This work can be extended to introduce all aforesaid uncertainties for the channel between controller and actuator.

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